Math 117 Sample Common Final Exam Questions

Section 1.2. Rate of Change

1. Find the average rate of change of $f(x) = 3x^2 + 1$ between the points.

   (a) $(1, 4)$ and $(2, 13)$

   (b) (j, k) and (m, n)

   (c) $(x, f(x))$ for $(x + h, f(x + h))$

2. The average rate of change of a function $T = f(x)$ on an interval is

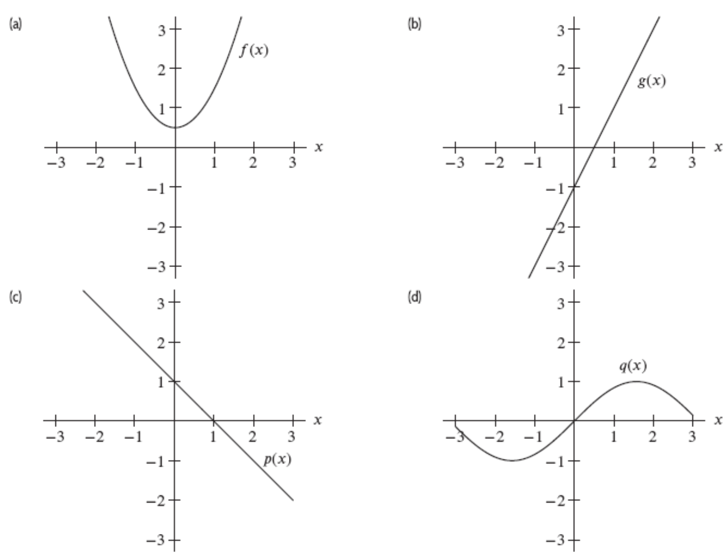
   (a) only $\frac{\Delta T}{\Delta x}$.

   (b) only $\frac{Rise}{Run}$.

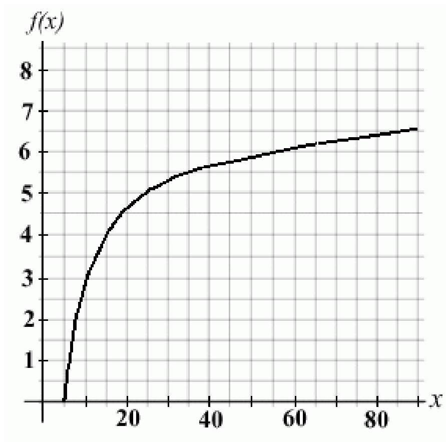
   (c) slope $\frac{f(b)-f(a)}{b-a}$.

   (d) All of the above.

3. Which of the functions in the figure below has the greatest rate of change on the interval $-1 \leq x \leq 1$?



4. The graph of the function $y = f(x)$ is shown below. Estimate the average rate of change of $f(x)$ between $x = 25$ and $x = 90$. Round your answer to two decimal places.



Section 1.3. Linear Functions

1. Could the data represent a linear function? If so, give the rate of change.

(a)

x	0	5	10	15
$f(x)$	10	20	30	40

(b)

x	0	10	20	30
$f(x)$	20	40	50	55

2. Identify the vertical intercept and the slope, and explain their meanings in practical terms.

   (a) The population of a town can be represented by the formula $P(t) = 54.25 - \frac{2}{7}t$, where $P(t)$ represents the population, in thousands, and t represents the time, in years, since 1970.

   (b) A stalactite grows according to the formula $L(t) = 17.75 + \frac{1}{250}t$, where $L(t)$ represents the length of the stalactite, in inches, and t represents the time, in years, since the stalactite was first measured.

3. A company finds that there is a linear relationship between the amount of money that it spends on advertising and the number of units it sells. If it spends no money on advertising, it sells 300 units. For each additional \$5000 spent, an additional 20 units are sold.

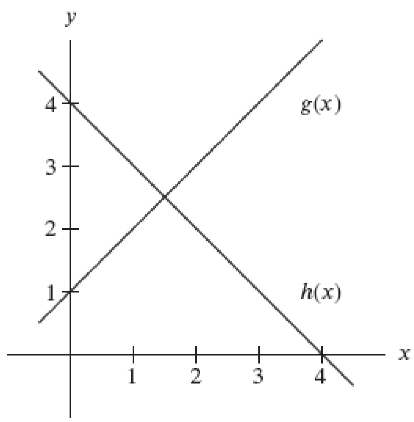
   (a) If x is the amount of money that the company spends on advertising, find a formula for y, the number of units sold as a function of x.

   (b) How many units does the firm sell if it spends \$25,000 on advertising? \$50,000?

   (c) How much advertising money must be spent to sell 700 units?

(d) What is the slope of the line you found in part (a)? Give an interpretation of the slope that relates units sold and advertising costs.

4. Use the figure above to find x- and y-intercepts of $f(x) = h(x) - g(x)$.



Section 1.4. Formulas for Linear Functions

1. Find a formula for the linear function satisfying the following conditions.

   (a) Slope -4 and x-intercept 7.

   (b) Passes through the points $(-1, 5)$ and $(2, -1)$.

   (c) Slope $2/3$ and passes through the point $(5, 7)$.

   (d) Has x-intercept 3 and y-intercept -5.

   (e) Function f has $f(0.3) = 0.8$ and $f(0.8) = -0.4$.

   (f) The graph of h intersects the graph of $y = x^2$ at $x = -2$ and $x = 3$.

2. Find the equations of the lines parallel to and perpendicular to the line $y + 4x = 7$, and through the point $(1, 5)$.

3. Table below gives the cost, $C(n)$, of producing a certain good as a linear function of n, the number of units produced.

n (units)	100	125	150	175
$C(n)$ (dollars)	11000	11125	11250	11375

   (a) $C(175)$

   (b) $C(175) - C(150)$

   (c) $\frac{C(175)-C(150)}{175-150}$

(d) Estimate $C(0)$. What is the economic significance of this value?

(e) The fixed cost of production is the cost incurred before any goods are produced. The unit cost is the cost of producing an additional unit. Find a formula for $C(n)$ in terms of n, given that

Total cost = Fixed cost + Unit cost · Number of units.

4. The demand for gasoline can be modeled as a linear function of price. If the price of gasoline is $p = \$3.10$ per gallon, the quantity demanded in a fixed period is $q = 65$ gallons. If the price rises to \$3.50 per gallon, the quantity demanded falls to 45 gallons in that period.

(a) Find a formula for q in terms of p.

(b) Explain the economic significance of the slope of your formula.

(c) Explain the economic significance of the q -axis and p -axis intercepts.

5. The table below gives data from a linear function. Find a formula for the function and determine the missing values.

x	-3	0		4	7	
$f(x)$	17		1	-18		-30

6. Using the data in the table below identify pairs of linear functions that are perpendicular.

x	0	1	2	3
$r(x)$	1/2	5/2	9/2	13/2
$s(x)$	1	−1	−3	−5
$t(x)$	1	1/2	0	−1/2
$u(x)$	5	7	9	11

7. What value of a makes the lines $y = 10 - \frac{2}{a}x$ and $y = -3 + 3x$ perpendicular?

8. If $x + 2y – 5 = 0$ then an equation of the same line is

(a) $y = -\frac{1}{2}x + \frac{5}{2}$ and only

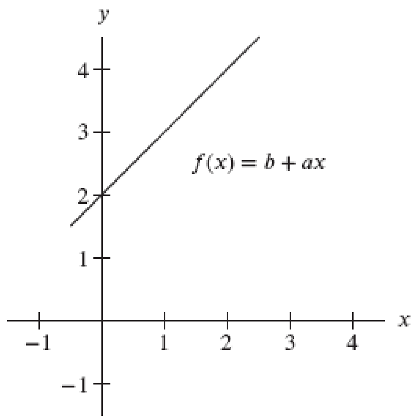
(b) $3x + 6y = 15$ and only

(c) $y - 0 = -\frac{1}{2}(x - 5)$ and only

(d) all of the above

(e) none of the above

9. The figure above shows the graph of $f(x) = b + ax$. Determine a and b.



Section 1.5. Modeling with Linear Functions.

1. Without a calculator, match the equations to the graphs

   (a) $y = x - 5$

   (b) $-3x + 4 = y$

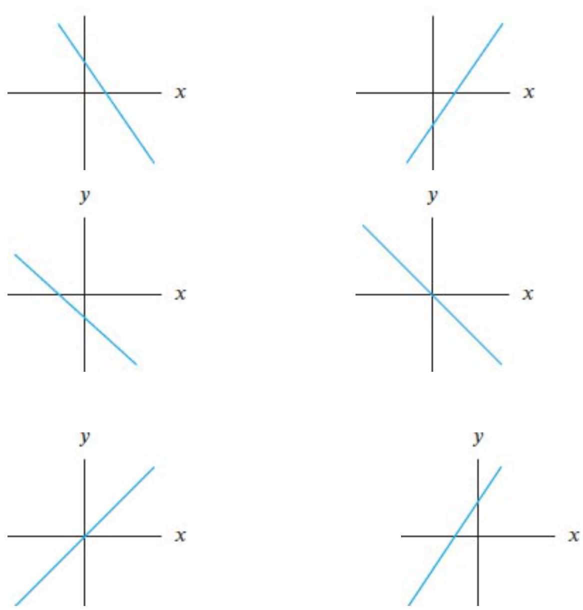
   (c) $5 = y$

   (d) $y = -4x - 5$

   (e) $y = x + 6$

   (f) $y = x/2$

   (g) $5 = x$



2. The cost of a Frigbox refrigerator is \$950, and it depreciates \$50 each year. The cost of an Arctic Air refrigerator is \$1200, and it depreciates \$100 per year.

   (a) If a Frigbox and an Arctic Air are bought at the same time, when do the two refrigerators have equal value?

   (b) If both refrigerators continue to depreciate at the same rates, what happens to the values of the refrigerators in 20 years' time? What does this mean?

3. In economics, the demand for a product is the amount of that product that consumers are willing to buy at a given price. The quantity demanded of a product usually decreases if the price of that product increases. Suppose that a company believes there is a linear relationship between the demand for its product and its price. The company knows that when the price of its product was \$3 per unit, the quantity demanded weekly was 500 units, and that when the unit price was raised to \$4, the quantity demanded weekly dropped to 300 units. Let D represent the quantity demanded weekly at a unit price of p dollars.

   (a) Calculate D when $p = 5$. Interpret your result.

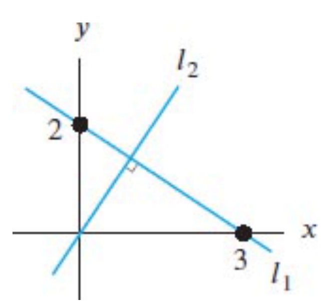
   (b) Find a formula for D in terms of p.

   (c) The company raises the price of the good and the new quantity demanded weekly is 50 units. What is the new price?

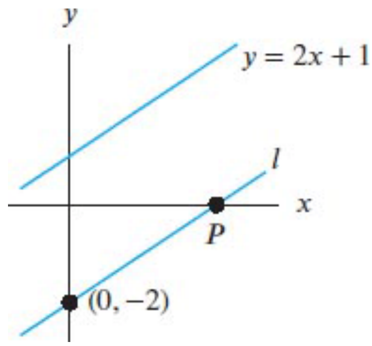
   (d) Give an economic interpretation of the slope of the function you found in part (b).

   (e) Find D when $p = 0$. Find p when $D = 0$. Give economic interpretations of both these results.

4. Find the equation of the line l_2 in the Figure below.



5. Line l in Figure is parallel to the line $y = 2x + 1$. Find the coordinates of the point P.

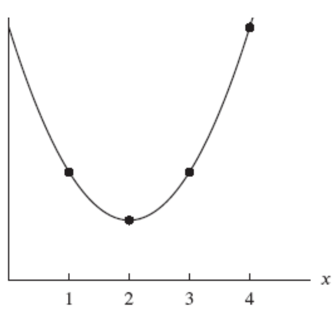


Section 2.1 Input and Output

1. Let $s(t) = 11t^2 + t + 100$ be the position, in miles, of a car driving on a straight road at time t, in hours. The car's velocity at any time t is given by $v(t) = 22t + 1$.
   (a) Use function notation to express the car's position after 2 hours. Where is the car then?
   (b) Use function notation to express the question, "When is the car going 65 mph?"
   (c) Where is the car when it is going 67 mph?

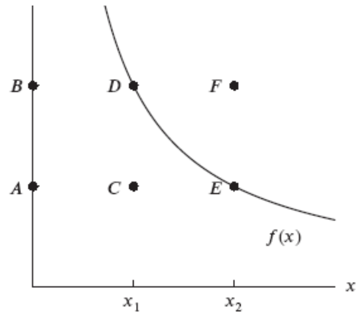
2. The profit, in dollars, made by a theater when n tickets are sold is $P(n) = 20n - 500$.
   (a) Calculate $P(0)$, and explain what this number means for the theater.
   (b) Under what circumstances will the profit equal 0?
   (c) What is the meaning of the quantity $P(100)$? What are its units?

3. From the graph of $s(x)$ in the figure below determine whether each expression is positive, negative or zero.
   (a) $s(2) - s(1)$
   (b) $s(3) - s(1)$
   (c) $s(4) - s(3)$
   (d) $s(1) - s(4)$



4. Match the expressions to the lettered points in the figure below.

   (a) $f(x_1)$
   (b) $f(x_2)$
   (c) $(x_1, f(x_1))$
   (d) $(x_2, f(x_2))$
   (e) $(x_1, f(x_2))$



5. Let $g(x) = x^2 + x$. Find formulas for the following functions. Simplify your answers.

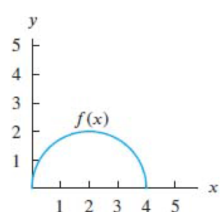
   (a) $g(-25x)$
   (b) $g(25 - x)$
   (c) $g(x + \pi)$
   (d) $g(\sqrt{x})$
   (e) $g(\frac{9}{x+1})$
   (f) $g(x^2)$

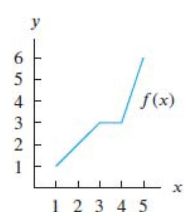
6. Let $k(x) = 8 - x^2$.

   (a) Find a point on the graph of $k(x)$ whose x -coordinate is -2.
   (b) Find two points on the graph whose y -coordinates are -24. Enter the exact answers in increasing order of x-coordinates.

Section 2.2 Domain and Range

1. Estimate the domain and range of the function. Assume the entire graph is shown.





2. Find the domain of the function algebraically.

   (a) $p(t) = \frac{1}{t^2-4}$

   (b) $f(x) = \frac{1}{\sqrt{9+x}}$

   (c) $f(x) = \sqrt{x^2 - 36}$

3. Let t be time in seconds and let $r(t)$ be the rate, in gallons/second, that water enters a reservoir: $r(t) = 800 - 4 \cdot t$

   (a) Evaluate the expressions $r(0)$, $r(15)$, $r(25)$, and explain their physical significance.

   (b) Graph $y = r(t)$ for $0 \le t \le 30$, labeling the intercepts. What is the physical significance of the slope and the intercepts?

   (c) For $0 \le t \le 30$, when does the reservoir have the most water? When does it have the least water?

   (d) What are the domain and range of $r(t)$?

4. A university was hit by a very contagious flu epidemic. It was determined that the number of people with the flu could be modeled by the function

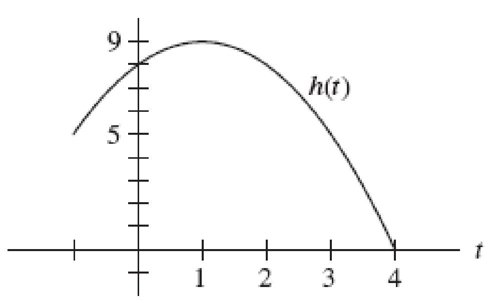
$$f(t) = \frac{2000}{1 + 19 \cdot (0.5)^t},$$

   where t represents the number of days.

   (a) What is the range of $f(t)$?

   (b) Evaluate $f(0)$, $f(5)$, and $f(10)$, and explain their meaning.

5. Find the domain and the range of $h(t)$ from the graph



Section 2.3 Piecewise-Defined Functions

1. Find the domain and range.

   (a)

$$G(x) = \begin{cases} x+1, & x < -1 \\ x^2+3, & x \geq -1 \end{cases} \tag{1}$$

   (b)

$$F(x) = \begin{cases} x^3, & x \leq 1 \\ 1/x, & x > 1 \end{cases} \tag{2}$$

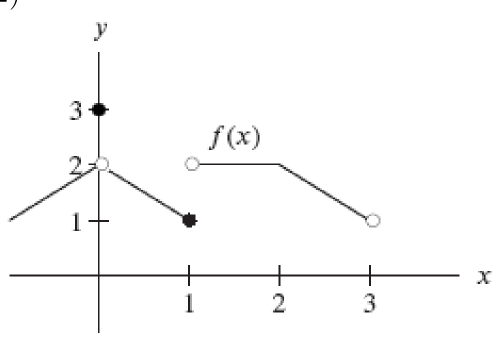
2. Let

$$g(x) = \begin{cases} -1, & x < 0 \\ x^3, & x \geq 0 \end{cases} \tag{3}$$

   (a) Find $g(-2)$, $g(2)$, $g(0)$.
   (b) Find the domain and range of $g(x)$.

3. Use the figure below to find

   (a) $f(3)$
   (b) $f(2)$
   (c) $f(1)$
   (d) $f(1/2)$
   (e) $f(0)$



Section 2.4 Preview of Transformations: Shifts

1. Using Table for $f(x)$ complete the tables for g, h, k, m, where:

   (a) $g(x) = f(x-1)$

   (b) $h(x) = f(x+1)$

   (c) $k(x) = f(x) + 3$

   (d) $m(x) = f(x-1) + 3$

   Explain how the graph of each function relates to the graph of $f(x)$

x	-2	-1	0	1	2
$f(x)$	-3	0	2	1	-1

x	-1	0	1	2	3
$g(x)$					

x	-3	-2	-1	0	1
$h(x)$					

x	-2	-1	0	1	2
$k(x)$					

x	-1	0	1	2	3
$m(x)$					

2. The domain of the function $g(x)$ is $-2 < x < 7$. What is the domain of $g(x-2)$?

3. The range of the function $R(s)$ is $100 \leq R(s) \leq 200$. What is the range of $R(s) - 150$?

4. The weight, V, of a particular baby named Jonah is related to the average weight function $s(t)$ by the equation $V = s(t) + 2$. Find Jonah's weight at ages $t = 3$ and $t = 6$ months. What can you say about Jonah's weight in general?

5. The graph of $g(x)$ contains the point $(-1, 2)$

   (a) Write a formula for a translation of g whose graph contains the point $(-1, 4)$.

   (b) Write a formula for a translation of g whose graph contains the point $(-3, 2)$.

6. Figure 1 shows the graph $y = f(x)$. Find a formula in terms of f for the graph of the function in Figure 2. Your formula should be of the form $y = f(x-h)+k$ for appropriate constants h and k. Identify h and k.

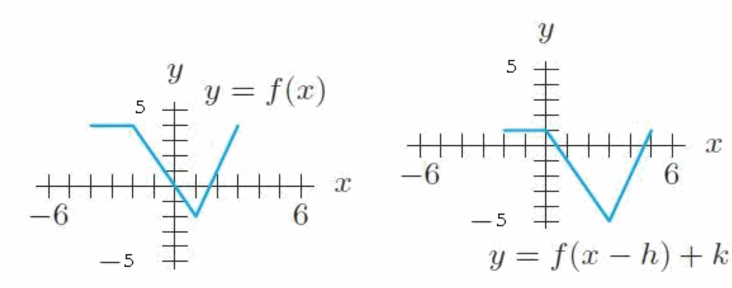


Figure 1   Figure 2

Section 2.5 Preview of Composite and Inverse Functions

1. Use $f(x) = 3x - 1$ and $g(x) = 1 - x^2$ to simplify the following expressions.

   (a) $f(g(0))$
   (b) $g(f(0))$
   (c) $g(f(2))$
   (d) $f(g(2))$
   (e) $f(g(x))$
   (f) $g(f(0))$
   (g) $f(f(x))$
   (h) $g(g(x))$

2. For $g(x) = 2x^3 - 1$ evaluate $g(\frac{3}{2})$ and $g^{-1}(-17)$.

3. The daily cost (in dollars) of renting a car and driving up to 500 miles a day is $C = f(m) = 32 + 0.19m$, where m is the number of miles driven.

   (a) Find the domain and range of $f^{-1}(C)$.
   (b) Find a formula for the inverse function $f^{-1}(C)$.

4. A circular oil slick is expanding with radius, $r$in yards, at time t in hours given by $r = 2t - 0.1t^2$ for $0 \le t \le 10$. Find a formula for the area in square yards, $A = f(t)$, as a function of time.

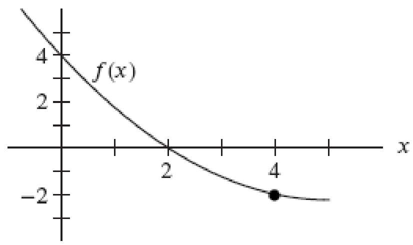
5. The cost, C, in thousands of dollars, of producing $q$kg of a chemical is given by $C = f(q) = 100 + 0.2q$. Find and interpret the following values.

   (a) $f(10)$
   (b) $f^{-1}(200)$
   (c) $f^{-1}(C)$

6. Determine if the given functions are inverses. $f(x) = -\frac{2}{x} - 1$; $g(x) = -\frac{2}{x+1}$.

7. Use the figure below to find the following values.

   (a) $f(0)$
   (b) $f^{-1}(-2)$
   (c) $f^{-1}(0)$
   (d) $f^{-1}(2)$
   (e) $f^{-1}(4)$
   (f) $f^{-1}(3)$
   (g) $f(f^{-1}(3))$



8. The formula for the volume of a cube with side s is $V = s^3$. The formula for the surface area of a cube is $A = 6s^2$.

   (a) Find and interpret the formula for the function $s = f(A)$.
   (b) If $V = g(s)$, find and interpret the formula for $g(f(A))$.

Section 2.6 Concavity

1. Do the graphs of the functions appear to be concave up, concave down, or neither?

   (a)

x	0	1	3	6
$f(x)$	1.0	1.3	1.7	2.2

   (b)

t	0	1	2	3	4
$g(t)$	20	10	6	3	1

   (c) $y = -x^2$
   (d) $y = x^3$, $x > 0$

2. Sketch a graph that is everywhere negative, increasing, and concave down.

3. Are the functions in problems below increasing or decreasing? What does the scenario tell you about the concavity of the graph modeling it?

   (a) When a drug is injected into a person's bloodstream, the amount of the drug present in the body increases rapidly at first. If the person receives daily injections, the body metabolizes the drug so that the amount of the drug present in the body continues to increase, but at a decreasing rate. Eventually, the quantity levels off at a saturation level.

(b) After a cup of hot chocolate is poured, the temperature cools off very rapidly at first, and then cools off more slowly, until the temperature of the hot chocolate eventually reaches room temperature.

4. Graph $f(x)$ with all of these properties:

   (a) $f(0) = 4$

   (b) f is decreasing and concave up for $-\infty < x < 0$

   (c) f is increasing and concave up for $0 < x < 6$

   (d) f is increasing and concave down for $6 < x < 8$

   (e) f is decreasing and concave down for $x > 8$

5. The graph of f is concave down for $0 \leq x \leq 6$. Which is bigger: $\frac{f(3)-f(1)}{3-1}$ or $\frac{f(5)-f(3)}{5-3}$? Why?

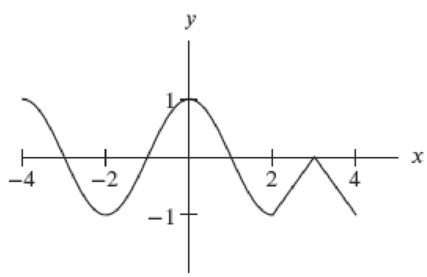
6. Use the figure to find the intervals where

   (a) the graph is concave up,

   (b) the graph is concave down,

   (c) neither concave up nor concave down,

   (d) parts concave up, parts concave down.



Section 3.1 Introduction to the Family of Quadratic Functions

1. Find the zeros (if any) of the function algebraically.

   (a) $y = (2 - x)(3 - 2x)$

   (b) $y = 9x^2 + 6x + 1$

   (c) $N(t) = t^2 - 7t + 10$

2. Find a formula for the quadratic function whose graph has the given properties.

   (a) A y -intercept of $y = 7$ and the only zero at $x = -2$.

   (b) A y -intercept of $y = 7$ and x -intercepts at $x = 1$, $x = 4$.

3. Using the factored form, find the formula for the parabola whose zeros are $x = -1$ and $x = 5$, and which passes through the point $(-2, 6)$.

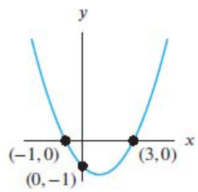
4. Let $V(t) = t^2 - 4t + 4$ represent the velocity after t seconds of an object in meters per second.

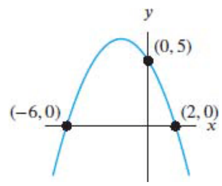
   (a) What is the object's initial velocity?

   (b) When is the object not moving?

   (c) Identify the concavity of the velocity graph.

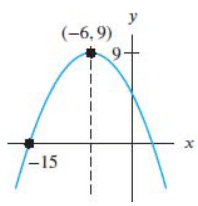
5. Find a formula for the parabola.

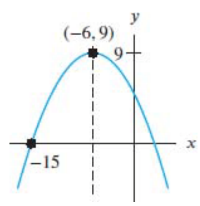


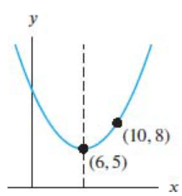


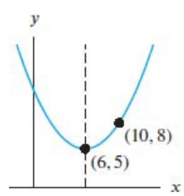
Section 3.2 The Vertex of a Parabola

1. Find a formula for the parabola.









2. Convert the quadratic functions to vertex form by completing the square. Identify the vertex and the axis of symmetry.

   (a) $f(x) = x^2 + 8x + 3$

   (b) $g(x) = -2x^2 + 12x + 4$

3. The parabola $y = ax^2 + k$ has vertex $(0, -2)$ and passes through the point $(3, 4)$. Find its equation.

4. Find a formula for the quadratic function whose graph has the given properties

   (a) Vertex at $(4, 2)$ and y-intercept of $y = -4$

   (b) Vertex at $(4, 2)$ and zeros at $x = -3, \quad 11$.

   (c) A single x -intercept at $x = 1/2$ and a y-intercept at 3.

Section 6.1 Shifts, Reflections, and Symmetry

1. The graph of $P = h(t)$ contains the point $(-6, -2)$.

   (a) If the graph has even symmetry, which other point must lie on the graph?

   (b) If the graph has odd symmetry, which other point must lie on the graph?

2. The function $Q(t)$ has domain $t \geq 0$ and a range of $-10 < Q(t) < 1$. Give the domain and range for the following transformations of $Q(t)$.

   (a) $y = Q(-t)$

   (b) $y = Q(t) - 6$

   (c) $y = -Q(-t)$

3. Are the functions below even, odd, or neither? Explain your reasoning.

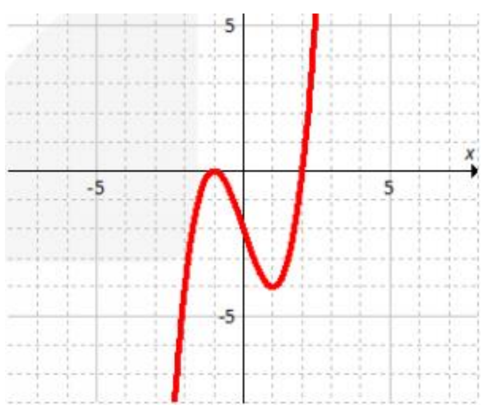
   (a) $m(x) = 3x^5$

   (b) $p(x) = x^3 - 1$

   (c) $q(x) = x^2 + 1$

Section 6.2 Shifts, Reflections, and Symmetry

1. Given the graph of $y = f(x)$ below, sketch the graph of $y = -2f(x + 3)$ on the same set of axes.



2. The function $y = r(t)$ has been transformed to create a new function $v(t) = \frac{1}{3}r(-2t)$. Explain each transformation in words.

3. The US population in millions is $P(t)$ today and t is in years. Write an expression in terms of $P(t)$ for each statement below.

   (a) The population 20 years before today.

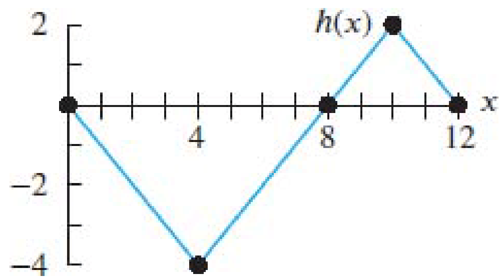
   (b) Today's population plus 8 million immigrants.

(c) Triple the population we have today.

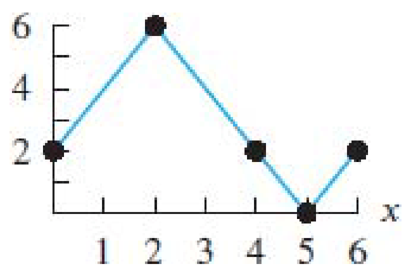
(d) The population after 1 million people have left the country.

Section 6.3 Horizontal Stretches and Combinations of Transformations

1. Consider the graph of $h(x)$



Find a formula for the following transformation of $h(x)$.



2. Given the function $4f(3t)-5$, describe the transformations that will be applied to $f(t)$ in the correct order.

3. The point $(8,-4)$ is on the graph of $y=m(x)$. Give the coordinates of one point of the graph of each of the following functions.

   (a) $y=m(\frac{1}{2}x)$

   (b) $y=\frac{1}{2}m(x)$

   (c) $y=m(-\frac{1}{2}x)$

   (d) $y=-m(2x)$

Section 11.1 Power Functions and Proportionality

1. Which of the following functions dominates the other as $x \to \infty$? Explain your reasoning.

$$y = 4x^{13} \qquad\qquad y = e^{2x}.$$

2. Suppose c is directly proportional to the square of t.
   (a) If $c = 75$ when $t = 5$, find the constant of proportionality.
   (b) Write a formula for c as a function of t.
   (c) Use your formula from part (b) to find c when $t = -4$.

3. (a) Describe the behavior of $y = -x^5$ as $x \to \infty$.
   (b) Describe the behavior of $y = x^4$ as $x \to \infty$.
   (c) Describe the behavior of $y = x^{-4}$ as $x \to \infty$.

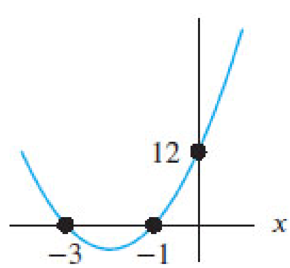
Section 11.2 Polynomial Functions

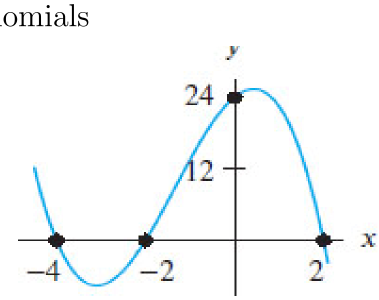
1. For the polynomials below, state the degree and the leading term, and describe the long-run behavior (i.e. as $x \to \infty$ and $x \to -\infty$).
   (a) $y = 3x^4 - x + 2$
   (b) $y = 4 - x^5 + 12x^2 + 3x$
   (c) $y = (5 - x)(2x + 1)$

2. Find the following limits.
   (a) $\lim_{x\to\infty} (x^2 - x)$
   (b) $\lim_{x\to-\infty} (1 - x - 4x^3)$
   (c) $\lim_{x\to\infty} (\frac{1}{5}x^4 - 2x^3 + 5)$

Section 11.3 The Short-Run Behavior of Polynomials

1. Find possible formulas for the following polynomials





2. Find a possible formula for each polynomial with the given properties:
   f has degree ≤ 2, $f(0) = 0$ and $f(1) = 1$.

3. Find all x-intercepts and y-intercepts of the following polynomials algebraically.

   (a) $f(x) = (x-1)(x+2)(x^2-16)$

   (b) $g(x) = x^3 - 5x^2 - 6x$

   (c) $h(x) = (x+4)(x^2 - 7x + 12)$

Section 11.4 Rational Functions

1. Find the following limits.

   (a) $$\lim_{n\to\infty} \frac{3n^2}{n^2+5}$$

   (b) $$\lim_{x\to-\infty} \frac{1}{(x-2)(x+1)}$$

   (c) $$\lim_{t\to\infty} \frac{t^3+2}{t-7}$$

2. Let t be the time in weeks. At time $t = 0$, organic waste is dumped into a pond. The oxygen level in the pond at time t is given by

   $$f(t) = \frac{t^2 - t + 1}{t^2 + 1}.$$

   Assume f(0)=1 is the normal level of oxygen.

   (a) What eventually happens to the oxygen level over large amounts of time?

   (b) Approximately how many weeks must pass before the oxygen returns to 75% of its normal level?

3. Find the horizontal asymptotes of the following functions if they exist.

   (a) $$p(x) = \frac{x}{x^3+8}$$

   (b) $$w(x) = \frac{(1-x)(3x+2)}{2x^2+5}$$

Section 11.5 The Short-Run Behavior of Rational Functions

1. Consider the rational function

   $$y = \frac{x+3}{x^2 - 2x - 8}.$$

   (a) What is the y-intercept?

   (b) Find any zeros of the function.

   (c) State the equations of any vertical asymptote.

2. Find a possible formula for the rational function in the figure below.