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Math 131 - Fall 2022 - Final Exam

Print name:____

Section number:__________ **Instructor's name:**______________________________

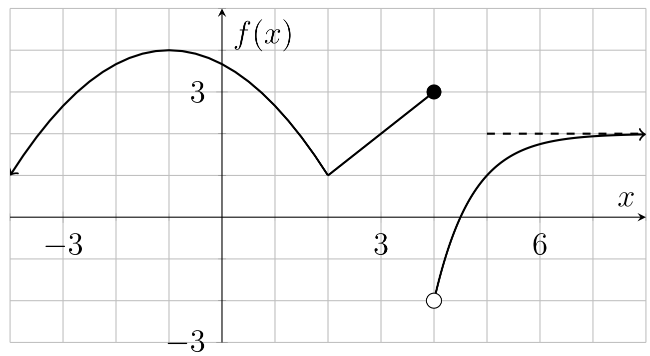
Directions:

- This exam has 11 questions.
- You will have two hours to complete this exam.
- It will be graded out of 100 points.
- Show your work. Answers (even correct ones) without the corresponding work will receive no credit.
- A formula sheet has been provided on the last page of this exam. You may not refer to any other notes or textbooks during the exam.
- You may use a calculator as long as it is unable to connect to the internet.
- You may only speak with your instructor during the exam.

Good luck!

Question:	1	2	3	4	5	6	7	8	9	10	11	Total
Points:	12	6	10	6	10	12	10	11	10	5	8	100
Score:												

1. [12 points] Using the graph below, find each of the following. If the answer does not exist, write "DNE". The dotted line on the right of the graph represents a horizontal asymptopte and is not a part of the function.



(a) [2 points] $f(2)$

(b) [2 points] $\lim_{x \to -1} f(x)$

(c) [2 points] $\lim_{x \to \infty} f(x)$

(d) [2 points] $\lim_{x \to 4^+} f(x)$

(e) [2 points] $\lim_{x \to 4^-} f(x)$

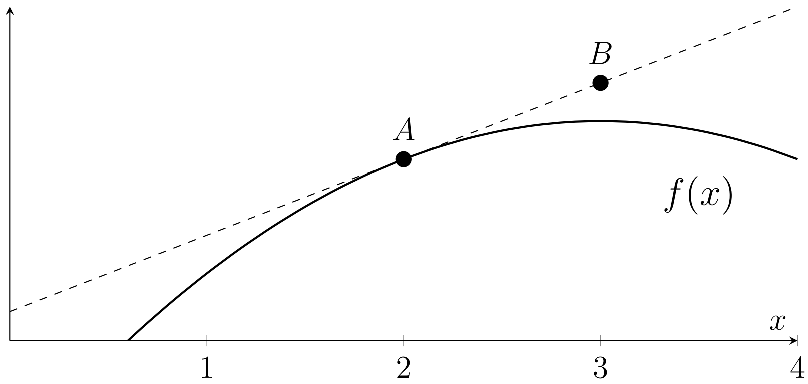
(f) [2 points] $\lim_{x \to 4} f(x)$

2. [6 points] Suppose $f(x) = x^2 - 5x + 1$.

(a) [3 points] Write the limit definition of $f'(4)$.

(b) [3 points] Evaluate the limit you found in part (a). *Do not use the power rule or other shortcuts!*

3. [10 points] The function in the figure has $f(2) = 5$ and $f'(2) = 2.1$.



(a) [4 points] Find the formula for the tangent line to $f(x)$ at $x = 2$.

(b) [3 points] Use the picture and your equation from part (a) to find the coordinates for point B. Present your answer as an (x, y) pair.

(c) [3 points] What is the slope of the tangent line for $h(x) = x \cdot f(x)$ at $x = 2$?

4. [6 points] A sports car goes from 0 mph to 60 mph in five seconds. Its velocity is given in the following table, converted from miles per hour to feet per second, so that all time measurements are in seconds.

Time, t (sec)	0	1	2	3	4	5
Velocity, $v(t)$ (ft/sec)	0	30	52	68	80	88

(a) [2 points] The **position** function is (circle one)

concave up **concave down** **constant**

(b) [4 points] Find the average **acceleration** between $t = 0$ and $t = 2$.

5. [10 points] Suppose $f(x) = \ln(\cos(x)) + 2^x$.

(a) [5 points] Find $f'(x)$.

(b) [5 points] Find $f''(x)$. *You do not need to simplify your final answer.*

6. [12 points] The gross domestic product (GDP) of a certain country during the recovery from a national crisis (at $t = 0$) is approximated by

$$G(t) = -0.4t^3 + 4.8t^2 + 20 \text{ for } 0 \leq t \leq 12,$$

where $G(t)$ is measured in billions of dollars and t is measured in years.

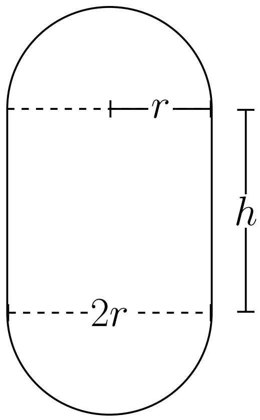
(a) [4 points] Find a formula for $G'(t)$ and interpret the expression $G'(3)$ in the context of this question. *Make sure to write your answer in a complete sentence with units.*

(b) [4 points] Find the critical points of $G(t)$ and classify them as minimums, maximums, or neither.

(c) [4 points] Identify any inflection points for this function on its domain.

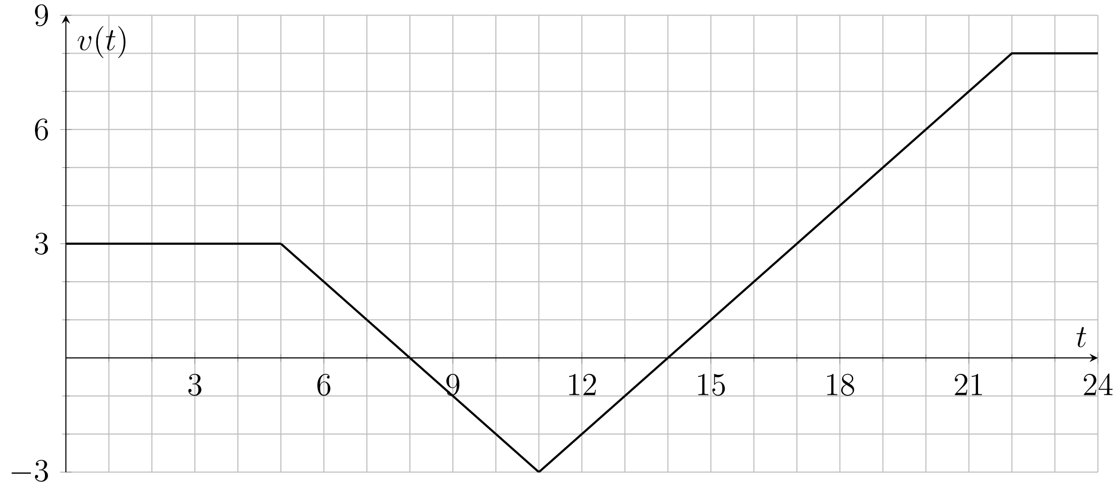
7. [10 points] A window has the shape of a rectangle with a semi circle on top and bottom; the diameter of the semicircles exactly match the width of the rectangle.

   (a) [5 points] Write an equation for the area of the window in terms of the radius r of the window's circular part given that the window has a perimeter of 10π inches.(Hint. The area of a circle is πr^2 and the circumference of a circle is $2\pi r$.)



   (b) [5 points] What is the maximum area the window can cover?

8. [11 points] A ship is moving out to sea directly away from (and sometimes directly towards) the shore. This graph shows the velocity for the ship over a 24 hour period of time. Time is measured in hours and the velocity is measured in km per hour.



(a) [5 points] Complete the table for the distance the ship is from shore after t hours.

Time (hrs)	0	5	11	14	22	24
Distance (km)	0					

(b) [2 points] What is the **average value** of the velocity of the ship over the first 14 hours of the trip?

(c) [2 points] On what time interval(s) is the distance from the shore decreasing?

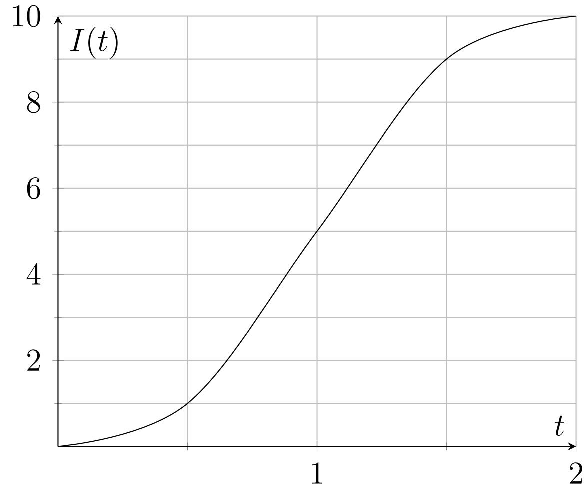
(d) [2 points] In the **first 12 hours** of the trip when is the ship a maximum distance from the shore?

9. [10 points] (a) [5 points] Compute the indefinite integral $\int \left(7 - 3e^x + \frac{4}{x}\right)\,dx$

(b) [5 points] Find the antiderivative $F(x)$ for $f(x) = x^4 + 6x - \sin(x)$ that satisfies the property $F(0) = 3$.

10. [5 points] Find the value of $\int_0^6 (f(x) + 3 \cdot g(x))\,dx$ given $\int_6^0 f(x)\,dx = 5$, and $\int_0^6 g(x)\,dx = 2$.

11. [8 points] The figure below shows the amount of electrical current $I(t)$, measured in coulombs per second, flowing through a wire t seconds after a switch is flipped.



(a) [5 points] Estimate $\int_0^2 I(t)\,dt$ using a right Riemann sum with 4 subdivisions.

(b) [3 points] Interpret $\int_0^2 I(t)\,dt$ in the context of this question. *Make sure to write your answer in a complete sentence with units.*

Five derivative rules for operations on functions.

Constant Multiple Rule: $\frac{d}{dx}[cf(x)] = cf'(x)$

Sum and Difference Rule: $\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$

Product Rule: $\frac{d}{dx}[f(x) \cdot g(x)] = f'(x)g(x) + f(x)g'(x)$

Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

Chain Rule: $\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$

Ten derivative rules for functions

Derivative of a Constant: $\frac{d}{dx}[c] = 0$, where c is a constant.

The Power Rule: $\frac{d}{dx}[x^n] = nx^{n-1}$

Exponential Functions: General Case: $\frac{d}{dx}[a^x] = a^x \cdot \ln(a)$

Exponential Functions: Special Case: $\frac{d}{dx}[e^x] = e^x$

Three Trigonometric Rules. $\frac{d}{dx}[\sin(x)] = \cos(x)$

$\frac{d}{dx}[\cos(x)] = -\sin(x)$

$\frac{d}{dx}[\tan(x)] = \sec^2(x) = \frac{1}{\cos^2(x)}$

Three Inverse Function Rules

$\frac{d}{dx}[\ln(x)] = \frac{1}{x}$

$\frac{d}{dx}[\arctan(x)] = \frac{1}{1+x^2}$

$\frac{d}{dx}[\arcsin(x)] = \frac{1}{\sqrt{1-x^2}}$

General Antiderivative Rules

If k is a constant $\int k\,dx = kx + C$

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$, when $n \neq -1$

$\int a^x\,dx = \frac{a^x}{\ln(a)} + C$

$\int e^x\,dx = e^x + C$

$\int \cos(x)\,dx = \sin(x) + C$

$\int \sin(x)\,dx = -\cos(x) + C$

$\int \sec^2(x)\,dx = \tan(x) + C$

$\int \frac{1}{x}\,dx = \ln(|x|) + C$

$\int \frac{1}{1+x^2}\,dx = \arctan(x) + C$

$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin(x) + C$