Common Final Exam

Calculus I, Math 161, Fall 2021

Name: ______________________________

Instructor's Name: ______________________________

Did you have another exam today 12/15 from 5:30 - 7:30? Circle YES or NO

Question	Points	Score
1	15	
2	20	
3	10	
4	10	
5	15	
6	5	
7	10	
8	15	
9	15	
10	15	
11	10	
12	10	
Total:	150	

- No books or notes of any kind are allowed.
- No technology – calculators, cell phones, or computers – is allowed.
- Show your work!
- You have 120 minutes to complete this exam.

1. Evaluate the following limits.

   (a) (5 points) $\displaystyle\lim_{x\to 2} \frac{x^2-2}{x^3-3x+5}$

   (b) (5 points) $\displaystyle\lim_{x\to 1} \ln(x^2)\left(\frac{1}{x-1}\right)$

   (c) (5 points) $\displaystyle\lim_{x\to 0} \frac{x-\sin x}{x^3}$

2. Calculate $\dfrac{dy}{dx}$

   (a) (5 points) $y = (x^4 - x)^5$

   (b) (5 points) $y = x\sin x$

   (c) (5 points) $y = (e^{2x} - 1)^8$

   (d) (5 points) $x^3 + y^3 = xy^2$

3. (10 points) Let $f(x) = \sqrt{25 - x^2}$. Find the equation of the line $L(x)$ which is tangent to $y = f(x)$ when $x = 3$.

4. (10 points) Find the coordinates of all inflection points of the function $f(x) = 3x^5 - 5x^4 - 2x + 1$. Justify your answer.

5. You own a large sports apparel company, selling Rambler t-shirts. Accounting for all related costs and revenue, the profit in thousands of dollars is given by

$$P(b) = -4 + 6b - b^2.$$

where b represents the number of boxes, measured in thousands, of t-shirts produced.

(a) (5 points) What is the net change of profit of the company if production increases from $b = 2$ to $b = 3$? Use appropriate units in your answer.

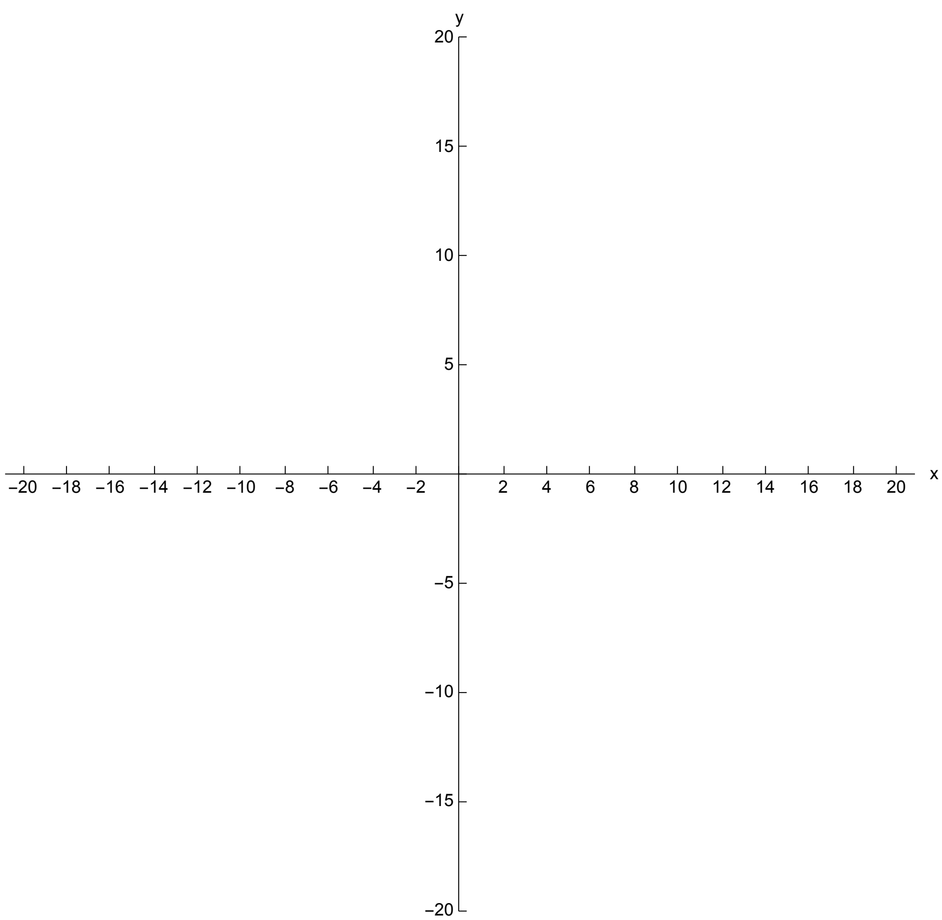
(b) (5 points) What is the instantaneous rate of change of profit with respect to b when $b = 2$? Use appropriate units in your answer.

(c) (5 points) Based on your answer in Part (b), you can advise the printshop manager to: increase production, decrease production, or leave the production at $b = 2$. What would you advise? Justify your answer.

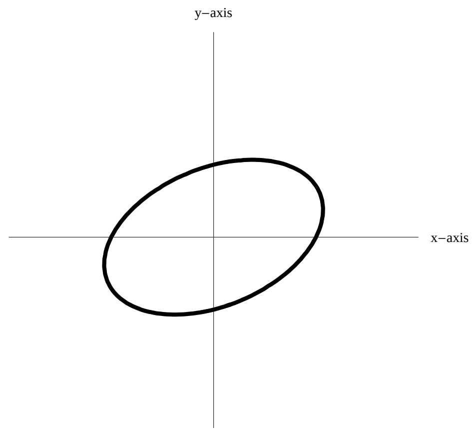
6. (5 points) Verify that the function $F(x) = x \ln x - x$ is an antiderivative of $f(x) = \ln x$.

7. (10 points) On the axes below, carefully sketch the graph of a single function $f(x)$ which has all of the following properties:

- $f(-8) = 5$ and $f(8) = 5$
- $\lim_{x \to 0} f(x) = +\infty$
- $\lim_{x \to -\infty} f(x) = 0$
- $\lim_{x \to +\infty} f(x) = 10$
- $f'(x) < 0$ on $(-8, -4)$ and $(0, 8)$
- $f'(x) > 0$ on $(-\infty, -8)$, $(-4, 0)$ and $(8, +\infty)$



8. The graph of the equation $x^2 - xy + 2y^2 = 4$ is an ellipse which is shown in the graph



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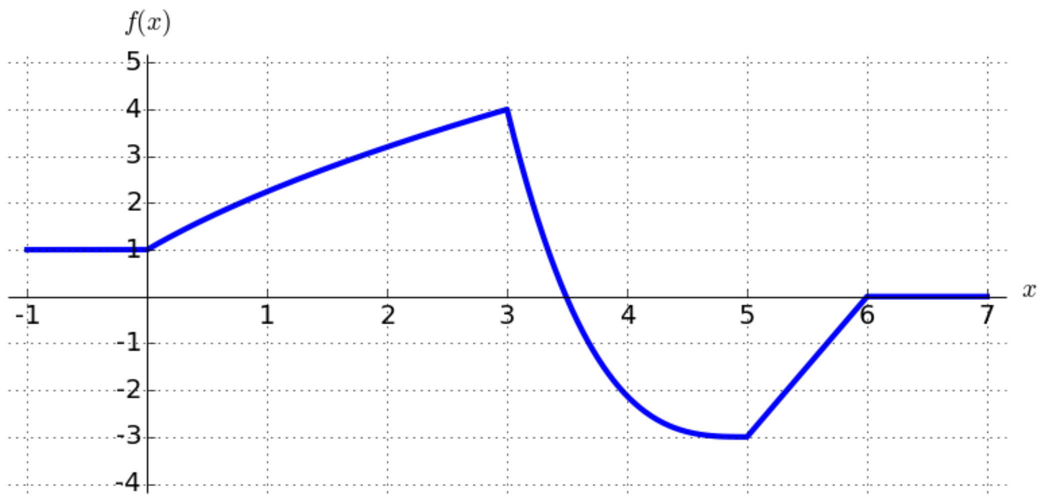
The derivative is given by

$$\frac{dy}{dx} = \frac{y - 2x}{4y - x}.$$

(a) (5 points) The ellipse crosses the x−axis twice. Find the coordinates of these two points.

(b) (10 points) Find the coordinates of all points on the ellipse where the tangent line is horizontal.

9. For the function f below, assume $-1 \leq a < b$ and consider $A = \displaystyle\int_a^b f(x)dx$.



(a) (5 points) What values (if any) of a and b minimize the value of A?

(b) (5 points) What values (if any) of a and b maximize the value of A?

(c) (5 points) Now take $a = 0$ and $b = 4$. Which is larger, a left-hand Riemann sum approximation of A or a right-hand Riemann sum approximation? (Assume four equal subintervals for each.)

10. Evaluate the following integrals.

(a) (5 points)

$$\int x^2 + \frac{1}{x}\, dx$$

(b) (5 points)

$$\int 2x\sqrt{1+x^2}\, dx$$

(c) (5 points)

$$\int_1^3 (x^2+1)^2\, dx$$

11. (10 points) A rectangular playground is to be constructed next to a building. The playground is to be enclosed with a fence, but the side next to the building does not need any fence. If you have 500 ft of fencing available, determine the dimensions of the playground that will have the largest area. A complete solution will indicate the domain of the function you are maximizing and why your answer is the maximum on that domain

12. (10 points) A 13 meter ladder is propped against a vertical wall. If the top of the ladder slides down the wall at a rate of 1 meter per minute and the top of the ladder is currently 12 meters high, at what rate does the base of the ladder slide away from the wall?