Math 118 Sample Common Final Exam Questions

Section 4.1 INTRODUCTION TO THE FAMILY OF EXPONENTIAL FUNCTIONS

1. Give the original price of an item and a percent increase or decrease. Compute the new price of the item.

   (a) \$40 and 10% increase

   (b) \$360 and 6.3% decrease

   (c) \$150 and 5.9% increase

   (d) \$40 and 10% decrease

2. Give the starting value a, the growth factor b, and the growth rate r if $Q = ab^t = a(1+r)^t$.

   (a) $Q = 1750(1.593)^t$

   (b) $Q = 34.3(0.788)^t$

3. What is the growth factor? Assume time is measured in the units given.

   (a) Water usage is increasing by 3% per year.

   (b) A diamond mine is depleted by 1% per day.

4. The value, V, of a $100,000$ investment that earns 3% annual interest is given by $V = f(t)$ where t is in years. How much is the investment worth in 3 years?

5. An investment decreases by 5% per year for 4 years. By what total percent does it decrease?

6. In 2014 the number of people infected by a virus was P_0. Due to a new vaccine, the number of infected people has decreased by 20% each year since 2014. In other words, only 80% as many people are infected each year as were infected the year before. Find a formula for $P = f(n)$, the number of infected people n years after 2014. Graph $f(n)$. Explain, in terms of the virus, why the graph has the shape it does.

Section 4.2. COMPARING EXPONENTIAL AND LINEAR FUNCTIONS

1. A population has size 5000 at time $t = 0$, with t in years.

   (a) If the population decreases by 100 people per year, find a formula for the population, P, at time t.

   (b) If the population decreases by 8% per year, find a formula for the population, P, at time t.

2. The tables contain values from an exponential or a linear function. In each problem:

   (a) Decide if the function is linear or exponential.

   (b) Find a possible formula for each function and graph it.

x	0	1	2	3	4
$f(x)$	12.5	13.75	15.125	16.638	18.301

x	0	1	2	3	4
$g(x)$	0	2	4	6	8

x	0	1	2	3	4
$h(x)$	14	12.6	11.34	10.206	9.185

x	0	1	2	3	4
$i(x)$	18	14	10	6	2

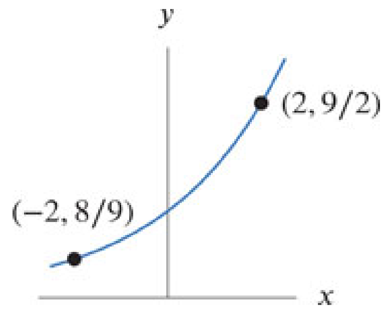
3. In year $t = 0$ a lake is estimated to have about 3500 trout in it. Ten years later, at $t = 10$, the population of trout is believed to be about 1700.

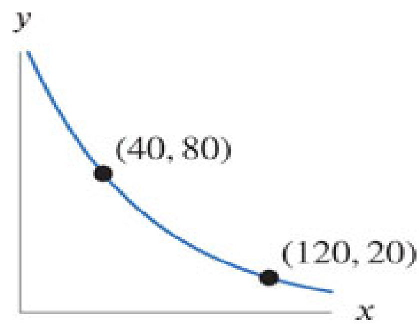
   (a) Write a formula for the size of the population P as a function of year t if we assume the decrease is linear. What is the rate of change, in fish per year, of the function over the ten-year period?

   (b) Write a formula for the size of the population P as a function of year t if we assume the decrease is exponential. What is the percent rate of change, in percent per year, of the function over the ten-year period?

   (c) Graph the two functions on the same coordinate system. Indicate the points at $t = 0$ and $t = 10$.

4. Find a formula for the exponential function.





5. According to the College Board the average cost of tuition (including fees), at private four-year nonprofit colleges rose from \$30,094 in 2013 to \$33,840 in 2016. What will the average cost of tuition be in 2020 assuming:

   (a) Linear growth?

   (b) Exponential growth?

Section 4.3. GRAPHS OF EXPONENTIAL FUNCTIONS

1. The population of Bulgaria was estimated to be 7.14 million in July 2016 and decreasing at 0.6% per year. If this trend continues:

   (a) Give a formula for the population, P, of Bulgaria, in millions, as a function of years, t, since 2016.

   (b) What is the predicted population in 2030?

   (c) By what percent is the population predicted to drop between 2016 and 2030?

   (d) Estimate t when the population is predicted to fall below 6 million.

2. The table shows the concentration of theophylline, a common asthma drug, in the blood stream as a function of time after injection of a 300-mg initial dose. It is claimed that this data set is consistent with an exponential decay model $C = ab^t$ where C is the concentration and t is the time.

Time (hours)	0	1	3	5	7	9
Concentration (mg/l)	12.0	10.0	7.0	5.0	3.5	2.5

   (a) Estimate the values of a and b, using ratios to estimate b. How good is this model?

   (b) Use a calculator or computer to find the exponential regression function for concentration as a function of time. Compare with your answers from part (a).

3. For which value(s) of a and b is $y = a\,b^x$ an increasing function? A decreasing function? Concave up?

4. What are the domain and range of the exponential function $Q = a\,b^t$ where a and b are both positive constants?

5. If $b > 1$, what is the horizontal asymptote of $y = a\,b^t$ as $t \to -\infty$?

6. If $0 < b < 1$, what is the horizontal asymptote of $y = a\,b^t$ as $t \to \infty$?

Section 4.4. APPLICATIONS TO COMPOUND INTEREST

1. Suppose 1000 is invested in an account yielding interest at a nominal rate of 8% per year. Find the balance three years later if the interest is compounded
   (a) Monthly
   (b) Weekly
   (c) Daily
2. An investment grows by 5% per year for 20 years. By what percent does it increase over the 20-year period?
3. An investment decreases by 60% over a 12-year period. At what effective annual percent rate does it decrease?
4. If you need $\$25,000$ six years from now, what is the minimum amount of money you need to deposit into a bank account that pays 5% annual interest, compounded:
   (a) Annually
   (b) Monthly
   (c) Daily
   (d) Your answers get smaller as the number of times of compounding increases. Why is this?
5. What are the nominal and effective annual rates for an account paying the stated annual interest, compounded annually? quarterly? daily?
   (a) 1%
   (b) 100%
   (c) 3%
   (d) 6%
6. What is the balance after 1 year if an account containing $500 earns the stated yearly nominal interest, compounded annually? weekly (52 weeks per year)? every minute (525,600 per year)?
   (a) 1%
   (b) 3%
   (c) 5%
   (d) 8%

Section 4.5. THE NUMBER e.

1. The expression shows how the quantity Q is changing over time t.
   (a) What is the quantity at time $t = 0$?
   (b) Is the quantity increasing or decreasing over time?
   (c) What is the percent per unit time growth or decay rate?
   (d) Is the growth rate continuous?
      i. $Q = 50 \cdot e^{1.05t}$
      ii. $Q = 2.7 \cdot 0.12^t$
      iii. $Q = 0.01 \cdot e^{-0.2t}$
      iv. $Q = 158 \cdot 1.137^t$
      v. $Q = 25 \cdot e^{0.032t}$
      vi. $Q = 2^t$

2. A town has population 3000 people at year $t = 0$. Write a formula for the population, P, in year t if the town
   (a) Grows by 200 people per year.
   (b) Grows by 6% per year.
   (c) Grows at a continuous rate of 6% per year.
   (d) Shrinks by 50 people per year.
   (e) Shrinks by 4% per year.
   (f) Shrinks at a continuous rate of 4% per year.

3. Find the effective annual yield and the continuous growth rate if $Q = 5500 \cdot e^{0.19t}$

4. A bank account pays 3% annual interest. As the number of compounding periods increases, the effective interest rate earned also rises.
   (a) Find the annual interest rate earned by the account if the interest is compounded:
      i. Quarterly
      ii. Monthly
      iii. Weekly
      iv. Daily
   (b) Evaluate $e^{0.03}$, where $e = 2.71828\ldots$. Explain what your result tells you about the bank account.

5. Three different investments are given.
   (a) Find the balance of each of the investments after a two-year period.
   (b) Rank them from best to worst in terms of rate of return. Explain your reasoning.

  i. Investment A: \$875 deposited at 13.5% per year compounded daily for 2 years.
  ii. Investment B: \$1000 deposited at 6.7% per year compounded continuously for 2 years.
  iii. Investment C: \$1050 deposited at 4.5% per year compounded monthly for 2 years.

6. During the first week of an epidemic, the number of people in a metropolitan area who are infected by the disease is growing at a continuous rate of 60% per day.

   (a) If 10 people are initially infected, how many people will be infected after a week has passed?

   (b) By what daily percentage does the number of infected people grow?

7. Which is larger after 5 years: an investment of \$1000 earning 5% per year compounded monthly or an investment of \$1100 earning 4% per year compounded continuously? Which is larger after 10 years? Justify your answers.

Section 5.1 LOGARITHMS AND THEIR PROPERTIES

1. Evaluate without a calculator.

   (a) $\log(1000)$

   (b) $\log(\sqrt{1000})$

   (c) $\log(1)$

   (d) $\log(0.1)$

   (e) $\log(10^0)$

   (f) $\log(\sqrt{10})$

   (g) $\log(10^5)$

   (h) $\log(10^{\log 100})$

   (i) $\log(10^{\log 0.1})$

   (j) $\ln(1)$

   (k) $\ln(e^5)$

   (l) $\ln(e^{\ln 2})$

   (m) $2\ln(e^4)$

   (n) $\ln(\frac{1}{\sqrt{e}})$

   (o) $\sqrt{\log(10,000)}$

2. Solve the equations exactly for x or t.

   (a) $3 \cdot 5^x = 9$

   (b) $4 \cdot 13^{3x} = 17$

   (c) $40 \cdot e^{-0.2t} = 12$

   (d) $200 \cdot 2^{t/5} = 355$

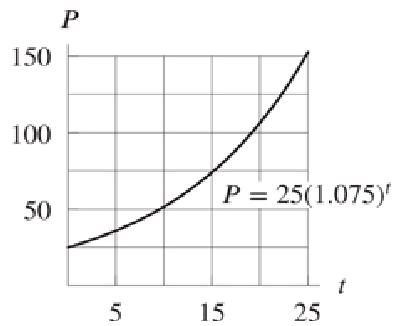
   (e) $3^{x+4} = 10$

   (f) $e^{x+5} = 7 \cdot 2^x$

   (g) $100^{2x+3} = \sqrt[3]{10,000}$

   (h) $0.4 \cdot (\frac{1}{3})^{3x} = 7 \cdot 2^{-x}$

3. A graph of $P = 25(1.075)^t$ is given below

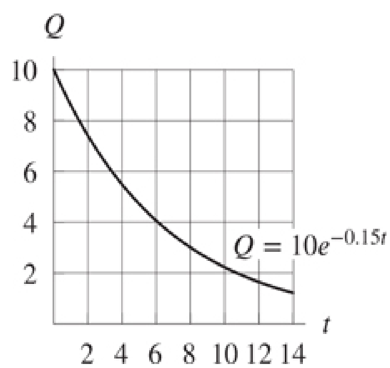


(a) What is the initial value of P (when $t = 0$)? What is the percent growth rate?

(b) Use the graph to estimate the value of t when $P = 100$.

(c) Use logs to find the value of t when $P = 100$.

4. A graph of $Q = 10 \cdot e^{-0.15t}$ is given below.



(a) What is the initial value of Q (when $t = 0$)? What is the continuous percent decay rate?

(b) Use the graph to estimate the value of t when $Q = 2$.

(c) Use logs to find the value of t when $Q = 2$.

Section 5.2 LOGARITHMS AND EXPONENTIAL MODELS

1. Solve for x.

   (a) $2 \cdot (1.3)^{-2x} = 14$

   (b) $5 \cdot (0.75)^{x} = 1.2$

   (c) $e^{7x} = 5 \cdot e^{3x}$

   (d) $\log(2x + 7) = 2$

2. Write the exponential function in the form $y = a\,b^t$. Find b accurate to four decimal places. If t is measured in years, give the percent annual growth or decay rate and the continuous percent growth or decay rate per year.

   (a) $y = 25 \cdot e^{0.053t}$

   (b) $y = 100 \cdot e^{-0.07t}$

3. Convert to the form $Q = a\,e^{kt}$.

   (a) $Q = 12 \cdot (0.9)^t$

   (b) $Q = 16 \cdot (0.487)^t$

   (c) $Q = 14 \cdot (0.862)^{1.4t}$

   (d) $Q = 721 \cdot (0.98)^{0.7t}$

4. Find the doubling time.

   (a) A bank account is growing by 2.7% per year.

   (b) A population is growing according to $P = P_0 \cdot e^{0.2t}$.

   (c) The population of a city is growing by 26% per year.

   (d) A company's profits are increasing by an annual growth factor of 1.12.

5. Find the half-life of the substance.

   (a) Einsteinium-253, which decays at a rate of 3.406% per day.

   (b) Tritium, which decays at a rate of 5.471% per year.

   (c) A radioactive substance that decays at a continuous rate of 11% per minute.

6. A population grows from 11000 to 13000 in three years. Assuming the growth is exponential, find the:

   (a) Annual growth rate

   (b) Continuous growth rate

   (c) Why are your answers to parts (a) and (b) different?

7. A population doubles in size every 15 years. Assuming exponential growth, find the
   (a) Annual growth rate
   (b) Continuous growth rate
8. A \$9000 investment earns 5.6% annual interest, and a \$4000 investment earns 8.3%, both compounded continuously. When will the smaller catch up to the larger?
9. What annual interest rate, compounded continuously, is equivalent to an annual rate of 8%, compounded annually?
10. What annual interest rate, compounded annually, is equivalent to an annual rate of 6%, compounded continuously?

Section 5.3 THE LOGARITHMIC FUNCTION AND ITS APPLICATIONS

1. What is the equation of the asymptote of the graph of $y = 10^x$? Of the graph of $y = 2^x$? Of the graph of $y = \log x$?

2. What is the equation for the asymptote of the graph of $y = e^x$? Of the graph of $y = e^{-x}$? Of the graph of $y = \ln(x)$?

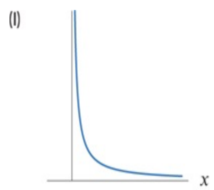
3. Match the statements (a)–(d) with one or more of the functions (I)–(IV).

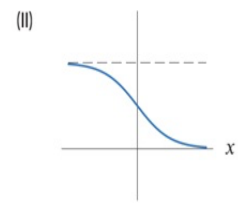
   (a) The graph has two horizontal asymptotes.

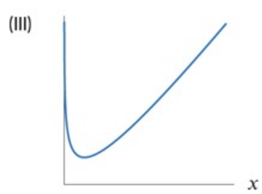
   (b) The graph has both a vertical asymptote and a horizontal asymptote.

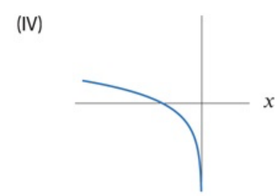
   (c) The graph tends towards infinity both as x tends to 0 and as x gets larger and larger.

   (d) The graph has a vertical asymptote at $x = 0$.









4. In chemistry, the acidity of a liquid is expressed using pH. The acidity depends on the hydrogen ion concentration in the liquid (in moles per liter); this concentration, written $[H+]$, varies in a small range between 0 and 1. The greater the hydrogen ion concentration, the more acidic the solution. The pH is defined as:

$$pH = -\log[H+]$$

. Find the hydrogen ion concentration, $[H+]$, for the substances.

   (a) Hydrochloric acid, with a pH of 0.

(b) Battery acid, with a pH of 1.

(c) Lye, with a pH of 13.

(d) Baking soda, with a pH of 8.3.

(e) Tomatoes, with a pH of 4.5.

5. Find the concentrations of hydrogen ions in solutions with

   (a) $pH = 2$

   (b) $pH = 4$

   (c) $pH = 7$

   A high concentration of hydrogen ions corresponds to an acidic solution. From your answers, decide if solutions with high pHs are more or less acidic than solutions with low pHs.

6. In an interview an oceanographer states that the seawater off the coast of Washington is projected to increase 150% in acidity by the end of the century. If we assume that the current average pH level is 8.1, find the projected pH level at the end of the century.

7. The hydrogen ion concentration of a stream with a population of rainbow trout was measured at $7.94(10^{-7})$. Rainbow trout begin to die at a pH value of 6. What percent increase in acidity in the stream water would cause the trout population to die? What would the corresponding change in pH value be?

8. Sound in decibels is measured by comparing the sound intensity, I, to a benchmark sound I_0 with intensity 10^{-16} $watts/cm^2$. Then, Noise level in decibels $= 10 \log{(I/I_0)}$.

   (a) Death of hearing tissue begins to occur at a noise level of 180 dB. Compute the sound's intensity at this noise level.

   (b) The noise level of a whisper is 30 dB. Compute the sound intensity of a whisper.

9. Use the Richter scale for the strength of an earthquake. The strength, W, of the seismic waves of an earthquake is compared to the strength, W_0, of the seismic waves of a standard earthquake. The Richter scale rating, M, is $M = log(W/W_0)$.

   (a) In 2017 the Belair earthquake near Washington, DC, had a Richter-scale rating of 4.1. How many times more powerful were the seismic waves of the Belair earthquake than standard seismic waves?

   (b) In 1986 the worst nuclear power plant accident in history occurred in Chernobyl, Ukraine. The explosion resulted in seismic waves with a Richter scale rating of 3.5. How many times stronger were the seismic waves of the Chernobyl disaster than standard seismic waves?

Section 11.6 COMPARING POWER, EXPONENTIAL, AND LOG FUNCTIONS.

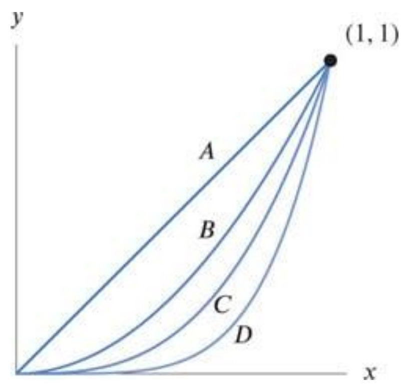
1. Without a calculator, match the following functions with the graphs.

   (a) $y = x^5$

   (b) $y = x^2$

   (c) $y = x$

   (d) $y = x^3$



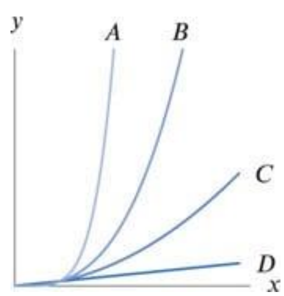
2. Without a calculator, match the following functions with the graphs

   (a) $y = x^5$

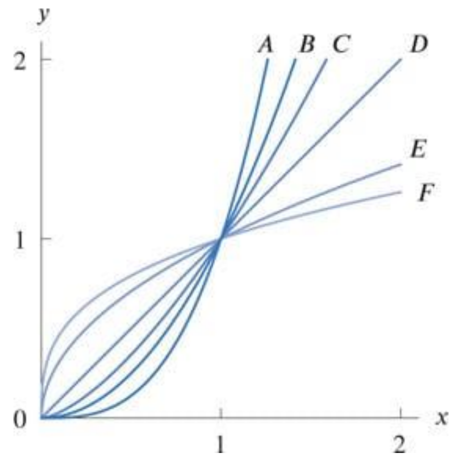
   (b) $y = x^2$

   (c) $y = x$

   (d) $y = x^3$



3. Match the functions x, x^2, x^3, $x^{1/2}$, $x^{1/3}$, $x^{3/2}$with the graphs



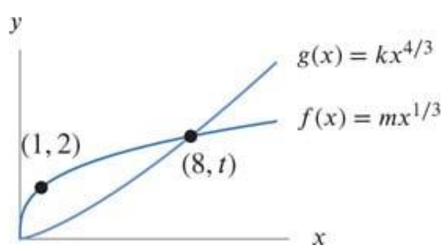
4. Let $f(x) = 4^x$ and $g(x) = x^2$

   (a) Complete the following table of values:

x	0	5	10	15	20	25	30
$f(x)$							
$g(x)$							

   (b) Describe the long-run behaviors of f and g as $x \to \infty$.

5. Find the values of m, t, and k.



6. The functions $y = x^{-3}$ and $y = 3^{-x}$ both approach zero as $x \to \infty$. Which function approaches zero faster? Support your conclusion numerically.

Section 7.1 INTRODUCTION TO PERIODIC FUNCTIONS

1. A Ferris wheel stands on a boarding platform 2 ft high and takes 10 minutes to make quarter of a revolution. After 30 minutes the rider is 65 feet above the ground. Find:

   (a) The maximum and the minimum height above the ground of a passenger riding the wheel.

   (b) The distance from the ground to the center of the wheel.

   (c) The radius of the wheel.

   (d) The time to complete a full revolution.

2. The graph describes your height, $h = f(t)$, above the ground on a Ferris wheel, where h is in meters and t is time in minutes. You boarded the wheel before $t = 0$. Determine the following:

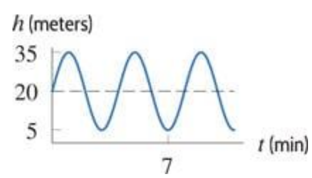
   (a) What is your height above the ground at $t = 0$?

   (b) Are you going up or down at $t = 0$?

   (c) How long does it take the wheel to complete one full revolution?

   (d) What is the diameter of the wheel?

   (e) At what height above the ground do you board the wheel? (The boarding platform is level with the bottom of the wheel.)



3. A weight is suspend from the ceiling by a spring. Let d be the distance in centimeters from the ceiling to the weight. When the weight is motionless, $d = 10$. If the weight is disturbed, it begins to bob up and down, or oscillate. Then d is a periodic function of t, time in seconds, so $d = f(t)$.

   Use the graph of f to determine

   (a) the midline,

   (b) period,

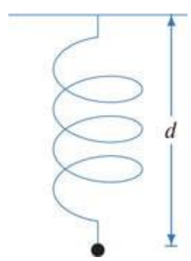
   (c) amplitude,

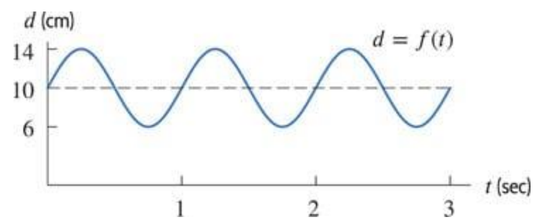
   (d) the minimum and

   (e) maximum values.

   (f) For the interval $0 < t < 0.25$ is the weight moving toward the ceiling or away from the ceiling?

   (g) For the interval $0.25 < t < 0.75$ is the weight moving toward the ceiling or away from the ceiling?



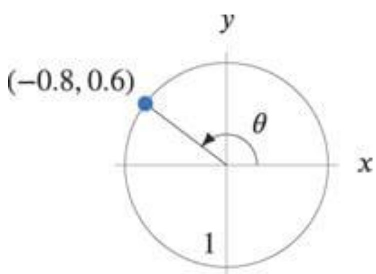


4. What is the period of the function? If it is not periodic, state that.

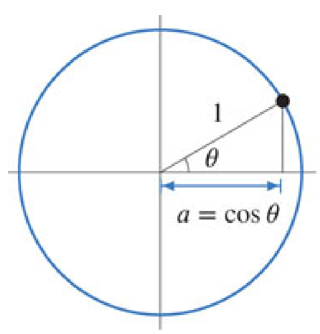
z	1	11	21	31	41	51	61	71	81
$g(z)$	5	3	2	3	5	3	2	3	5

Section 7.2 THE SINE AND COSINE FUNCTIONS

1. What angle (in degrees) corresponds to 3.25 rotations around the unit circle?
2. Find the coordinates of the point at $-270°$ on a circle of radius 3.8 centered at the origin.
3. Using the graph, for the angle θ
   (a) evaluate $\sin(\theta)$,
   (b) evaluate $\cos(\theta)$



4. Find all angles between $-360°$ and $360°$ (but not $53°$) that have the same cosine as $53°$.
5. Let θ be an angle in the first quadrant with $\cos(\theta) = a$. Evaluate the following in terms of a.



   (a) $\cos(\theta + 360°)$
   (b) $\cos(\theta + 180°)$
   (c) $\sin(90° - \theta)$
   (d) $\cos(180° - \theta)$

(e) $\cos(360^\circ - \theta)$

(f) $\sin(270^\circ - \theta)$

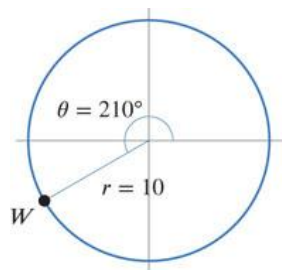
Section 7.3 RADIANS AND ARC LENGTH

1. What is the radian measure of the angle $100°$? Leave an exact answer.

2. Convert the angle 7 radians into degrees. Leave as an exact answer.

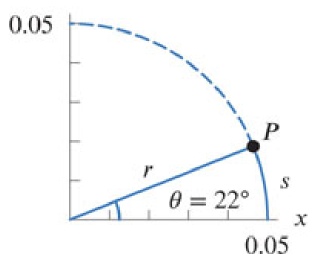
3. What angle in radians corresponds to 0.75 rotations around the unit circle?

4. Find exact values for the coordinates of point W.



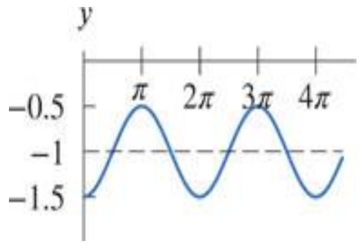
5. What is the angle determined by an arc of length 2π meters on a circle of radius 18 meters?

6. Where possible give the radius r, the measure of θ in both degrees and radians, the arc length s, and the coordinates of point P.



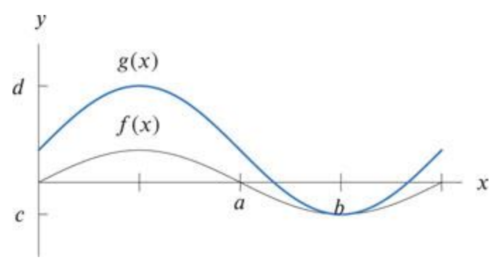
Section 7.4 GRAPHS OF SINE AND COSINE FUNCTIONS

1. Given the graph of the sinusoidal below, what is the
   (a) midline?
   (b) amplitude?
   (c) period?
   (d) Write the equation of the sinusoidal function in the form of $y = a\cos(bx) + d$ or $y = a\sin(bx) + d$.



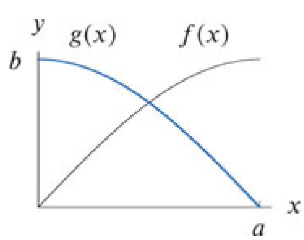
2. Find all possible formulas of the form $A\sin(t) + k$ or $A\cos(t) + k$ for the periodic functions with the given properties. (There may be just one; there may be more than one.) Midline $y = 3$, amplitude 2 with a maximum value at $t = 0$.

3. The figure below shows $y = \frac{1}{2}\sin x$ and $y = \sin x + \frac{1}{2}$.



   (a) Which graph, $f(x)$ or $g(x)$ is $y = \frac{1}{2}\sin x$?
   (b) Find the value for a.
   (c) Find the value for b.
   (d) Find the value for c.
   (e) Find the value for d.

4. The figure shows $y = \sin x$ and $y = \cos x$ starting at $x = 0$. Which is $y = \cos x$? Find values for a and b.



5. The height (in meters) of a person on a Ferris wheel above the ground as a function of the angle from the 3 o'clock position is given by $f(\theta) = 50 + 45\sin(\theta)$.

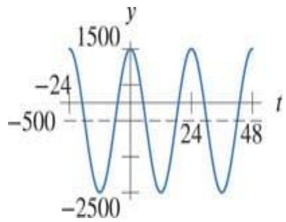
   (a) What is the highest point on the ride?

   (b) What is the lowest point on the ride?

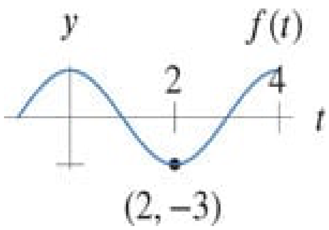
   (c) What is the diameter of the Ferris wheel?

Section 7.5 SINUSOIDAL FUNCTIONS

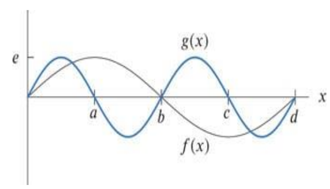
1. Estimate the period, midline, and amplitude of the periodic function.



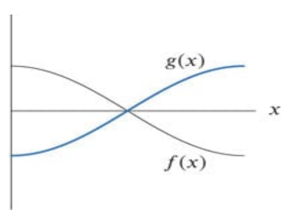
2. Find a formula for the trigonometric function



3. The figure shows $y = \sin x$ and $y = \sin 2x$. Which graph is $y = \sin x$? Identify the points a to e.



4. The figure is $y = \sin(x - 90°)$ and $y = \sin(x + 90°)$ starting at $x = 0$. Identify which is which.



5. A population of animals oscillates between a low of 1300 on January 1 ($t = 0$) and a high of 2200 on July 1 ($t = 6$).

(a) Find a formula for the population, P, in terms of the time, t, in months.

(b) Interpret the amplitude, period, and midline of the function $P = f(t)$.

(c) Use a graph to estimate when $P = 1500$.

6. Household electrical power in the US is provided in the form of alternating current. Typically the voltage cycles smoothly between $+155.6$ volts and -155.6 volts 60 times per second. Use a sinusoidal function to model voltage, V, as a function of time, t, in seconds since the voltage was at its peak.

Section 7.6 THE TANGENT FUNCTION

1. Evaluate without a calculator. Leave the exact answer.

   (a) $\tan(\pi/2)$

   (b) $\tan(\pi/3)$

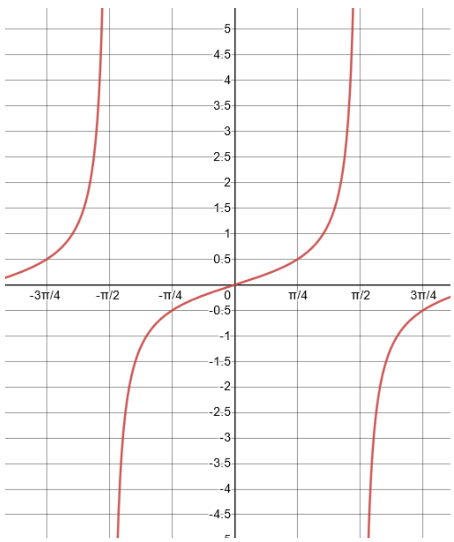
   (c) $\tan(\pi/4)$

   (d) $\tan(\pi/6)$

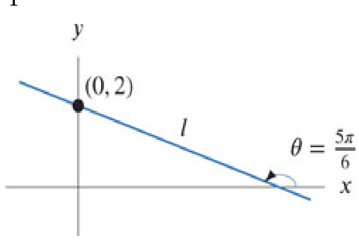
   (e) $\tan(0)$

2. Find the vertical asymptotes of the function $g(\theta) = \tan(2\theta)$ in the interval $0 \leq \theta \leq 2\pi$. (HINT: What does multiplying the input (an inside change) by 2 do to a function?)

3. Give a formula for the graph below.



4. Find an equation for the line l in the figure below. Find the x-intercept of the line.



Section 7.7 THE SIX TRIGONOMETRIC FUNCTIONS AND RELATIONSHIPS BETWEEN THEM

1. Find exact values without a calculator.

   (a) $\cot(\pi/4)$

   (b) $\csc(5\pi/3)$

2. Give exact answers for $0 \leq \theta \leq \pi/2$.

   (a) If $\cos(\theta) = 1/2$, what is $\csc(\theta)$? $\cot(\theta)$?

   (b) If $\sin(\theta) = 1/3$, what is $\sec(\theta)$? $\tan(\theta)$?

3. Given the expression $\sin(\theta) = x/3$ with θ in the first quadrant, find expressions for

   (a) $\cos(\theta)$

   (b) $\tan(\theta)$

   Your answers will be algebraic expressions in terms of x.

Section 7.8 INVERSE TRIGONOMETRIC FUNCTIONS

1. Evaluate each expression

   (a) $\sin(x)$ if $x = \frac{1}{2}^{\circ}$

   (b) $\sin^{-1}(x)$ if $x = \frac{1}{2}$

   (c) $(\sin(x))^{-1}$ if $x = \frac{1}{2}^{\circ}$

2. Solve the equation, $6\cos(\theta) - 2 = 3$, for a value of θ in the first quadrant. Give your answer in radians and degrees.

3. Solve the equation $\cos(t) = 1/2$ exactly on $0 \le t \le 2\pi$, for all t.

4. In this equation, $\sin(k) = a$,

   (a) the angle is (circle one): k, a, impossible to tell.

   (b) the value of the trigonometric function is (circle one): k, a, impossible to tell.

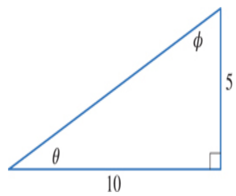
5. Without a calculator, evaluate the following exactly.

   (a) $\sin(\sin^{-1}(1/2))$

   (b) $\cos^{-1}(\cos(5\pi/3))$

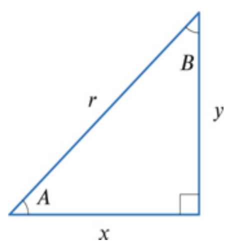
Section 8.1 TRIG FUNCTIONS AND RIGHT TRIANGLES

1. Find exactly:



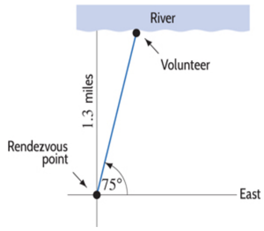
   (a) $\sin(\theta)$
   (b) $\sin(\phi)$
   (c) $\cos(\theta)$
   (d) $\cos(\phi)$
   (e) $\tan(\theta)$
   (f) $\tan(\phi)$

2. One of the sides x, y and r of the triangle in Figure below is given. Find exact values of the other two sides.



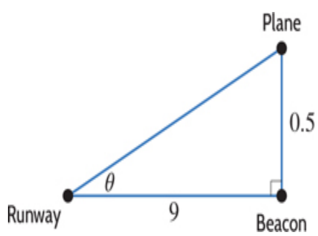
   (a) $A = 17°$, $b = 73°$, $r = 7$
   (b) $A = 12°$, $b = 78°$, $y = 4$
   (c) $A = 37°$, $b = 53°$, $x = 6$
   (d) $A = 40°$, $r = 15$
   (e) $B = 77°$, $x = 9$
   (f) $B = 22°$, $x = \lambda$

3. A search and rescue volunteer leaves a rendezvous point in the Arizona desert walking 75 degrees north of east. She reaches a river that runs east-west and is located 1.3 miles directly north of the rendezvous point. How far east of the rendezvous point is the volunteer when she reaches the river?



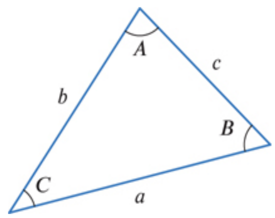
4. A 240-ft tree casts a 130-foot shadow on horizontal ground. A girl lying on the grass at the tip of the shadow is looking at a bird nesting at the top of the tree. At what angle is the girl looking up?

5. A plane flying at an altitude of 0.5 miles passes over a beacon that is located 9 miles from the end of a runway. The plane then starts its final approach to landing. Find the glide slope angle, θ, of the landing plane.



Section 8.2 NON-RIGHT TRIANGLES

1. Find the missing sides, a, b, c, and angles, A, B, C (if possible). If there are two solutions, find both.



   (a) $a = 20,\ b = 28,\ c = 41$
   (b) $a = 14,\ b = 12,\ C = 23°$
   (c) $a = 20,\ B = 81°,\ c = 28$
   (d) $a = 20,\ B = 28,\ C = 12°$
   (e) $a = 9,\ b = 8,\ C = 80°$
   (f) $A = 13°,\ B = 25°,\ c = 4$
   (g) $A = 12°,\ C = 150°,\ c = 5$

2. To measure the height of the Eiffel Tower in Paris, a person stands away from the base and measures the angle of elevation to the top of the tower to be 60°. Moving 210 feet closer, the angle of elevation to the top of the tower is 70°. How tall is the Eiffel Tower?

3. Two airplanes leave Kennedy Airport in New York at 11 am. The air traffic controller reports that they are traveling away from each other at an angle of 103°. The A380 travels 509 mph and the 787 travels at 503 mph. How far apart are they at 11:30 am?

4. A parcel of land is in the shape of an isosceles triangle. The base has length 425 feet; the other sides, which are of equal length, meet at an angle of 39°. How long are they?

5. In video games, images are drawn on the screen using xy-coordinates. The origin, $(0, 0)$, is the lower-left corner of the screen. An image of an animated character moves from its position at $(8, 5)$ through a distance of 12 units along a line at an angle of 25° to the horizontal. What are its new coordinates?

Section 9.1 TRIGONOMETRIC EQUATIONS

1. Use a graph to approximate solutions to the equation $\cos(t) = 0.4$ on $-\pi \leq t \leq 3\pi$.
2. Solve the equation $2\sqrt{3}\tan(10\theta) + 4 = 6$ for a value of θ in the first quadrant. Give your answer in radians and degrees.
3. Solve the equation $4\cos(2\theta) + 5 = 2\cos(2\theta) + 6$ for a value of θ in the first quadrant. Give your answer in radians and degrees.
4. Find all solutions to the equation $3\cos(x) = \frac{1}{\cos(x)}$ for $0 \leq x \leq 2\pi$.
5. Find exact values for all solutions to the equation $\sin(\theta) = \frac{\sqrt{3}}{2}$ for $-2\pi \leq \theta \leq 2\pi$.
6. Solve for x: $6\arcsin(3x) - 2\pi = 0$. If it is not possible, state why.
7. Solve for x: $3\arccos(2x - 5\pi) + 4 = 64$. If it is not possible, state why.
8. Find the exact value(s) of x that satisfies the equation on the interval $[0, 2\pi]$.
   (a) $\sec(2x + \pi) = 2$
   (b) $\cos\left(\frac{x}{3}\right) = \frac{1}{2}$

Section 9.2 IDENTITIES, EXPRESSIONS, AND EQUATIONS

1. Find all solutions for $0 \le \theta \le 2\pi$, give exact answers:

   (a) $2\tan^2(\theta) = 6$

   (b) $6\sin(\theta + \pi) = \sqrt{18}$

2. Assuming $\sin(x) = \frac{7}{6}$ and $\cos(x) = \frac{\sqrt{13}}{6}$. Find $\sin(2x)$, simplify and leave exact answers.

3. Let $\csc(\theta) = 18$. For $\frac{\pi}{2} \le \theta \le \pi$, find the following using identities, not a right triangle. Be sure to show all work and give exact answers.

   (a) $\sin(\theta) =$

   (b) $\cos(\theta) =$

   (c) $\tan(\theta) =$

   (d) $\sec(\theta) =$

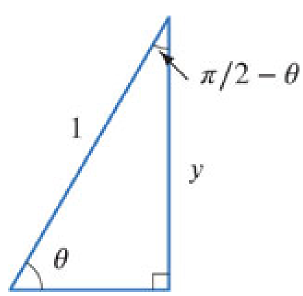
   (e) $\cot(\theta) =$

4. Determine if the following are true, it may be helpful to use the trigonometric identities:

   (a) $(\tan^2(x) + 1)(\cos^2(x)) + 3\tan(x)\cot(x) = 4$

   (b) $1 - 2\tan^2(\theta)\cos^2(\theta) = 2\cos^2(\theta) - 1$

5. With y and θ as in the figure below and with $0 < \theta < \pi/4$, express the following in terms of y without using trigonometric functions:



   (a) $\sin(\cos^{-1}(y))$

   (b) $\sin(2\theta)$

   (c) $\cos\left(\frac{\pi}{2} - \theta\right)$

   (d) $\tan^2\theta$

   (e) $\tan(2\theta)$

Section 9.3 SUM AND DIFFERENCE FORMULAS FOR SINE AND COSINE

1. Find the exact values of the following. Show your work.
   (a) $\csc\left(\frac{7\pi}{12}\right)$
   (b) $\cos(15°)$
   (c) $\tan\left(\frac{5\pi}{12}\right)$
2. Let $\sin(A) = 0.67, \cos(A) = 0.74, \sin(B) = 0.87$ and $\cos(B) = 0.48$. Find the following values.
   (a) $\sin(A + B)$
   (b) $\cos(A + B)$
   (c) Use your work from parts (a) and (b) to find $\tan(A + B)$.

Section 9.4 POLAR COORDINATES

1. Convert the Cartesian coordinates given below to polar coordinates.
   (a) $(1, 1)$
   (b) $(-1, 0)$
   (c) $(-\sqrt{3}, 1)$

2. Convert the polar coordinates given below to Cartesian coordinates. Give exact answers.
   (a) $(1, 2\pi/3)$
   (b) $(2\sqrt{3}, -\frac{\pi}{6})$
   (c) $(\sqrt{3}, -\frac{3\pi}{4})$

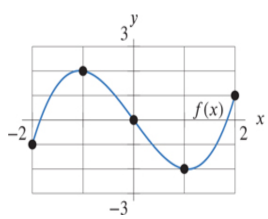
Section 10.1 REVISITING COMPOSITION OF FUNCTIONS

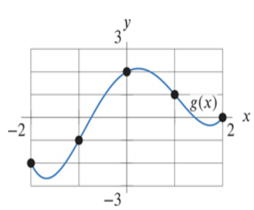
1. For $f(x) = -x^2 - x$, find and simplify

   (a) $f(1)$
   (b) $f(-2)$
   (c) $f(h)$
   (d) $f(-y)$
   (e) $f(y+1)$
   (f) $f(1-h)$

2. Use Figures to calculate the following:

   (a) $f(g(-1))$
   (b) $g(f(2))$
   (c) $f(g(-2))$
   (d) $g(f(0))$
   (e) $f(f(1))$
   (f) $g(g(g(-1)))$





3. Decompose the function into two new functions u, v, where v is the inside function, that is $f(x) = u(v(x))$, and $u(x) \neq x$, and $v(x) \neq x$.

   (a) $f(x) = 3\log(x) + 5$
   (b) $f(x) = \dfrac{e^{\sin x}}{\sin x}$

(c) $f(x) = 5\sqrt{x+3}$

(d) $f(x) = \dfrac{8}{x-11}$

(e) $f(x) = (x^3 - 18)^4 + 15$

4. A side effect of a therapeutic drug is raising the patient's heart rate. The relation between Q, the amount of drug in the patient's body (in milligrams), and r, the patient's heart rate (in beats per minute), is

$$r = f(Q) = 60 + 0.2Q$$

Over time, the level of the drug in the patient's bloodstream falls. The drug level as a function of time t, in hours since the initial injection, is

$$Q = g(t) = 250(0.8)^t$$

Find a formula for the heart rate r as a function of time t.

5. A software company issues a bug patch. After t days, the company estimates that the number of installed patches (in thousands) is

$$g(t) = \frac{60}{10 + 40(1/2)^{t/10}}$$

For $a = 10,\ h = 20$, evaluate $\frac{g(a+h)-g(a)}{h}$. What does it tell you about the adoption of the patch.

Section 10.2 REVISITING INVERSE FUNCTIONS

1. For each function $f(x)$ given below, find the inverse of $f(x)$. That is, find $f^{-1}(x)$. If no inverse exists, write "no inverse."

   (a) $f(x) = 10^{9x-2}$

   (b) $f(x) = 3x + 5$

   (c) $f(x) = 12x^2 + 6x - 15$

   (d) $f(x) = e^{4x}$

2. If $t = g(v)$ represents the time in hours it takes to drive to the next town at velocity v mph, what does $g^{-1}(t)$ represent? What are its units?

3. The balance B, in dollars, in an account after t years with an initial deposit of \$500 that pays 4% compounded annually is $B = f(t) = 500(1.04)^t$.

   (a) Find a formula for $t = f^{-1}(B)$.

   (b) What does the $f^{-1}(B)$ represent in terms of the account?

4. Let $P = f(t) = 37.8(1.044)^t$ be the population of a town (in thousands) in year t.

   (a) Describe the town's population in words.

   (b) Evaluate $f(50)$. What does this quantity tell you about the population?

   (c) Find a formula for $f^{-1}(P)$ in terms of P.

   (d) Evaluate $f^{-1}(50)$. What does this quantity tell you about the population?

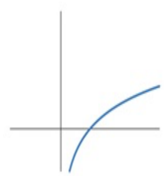
5. The noise level, N, of a sound in decibels is given by

   $$N = f(I) = 10 \log\left(\frac{I}{I_0}\right),$$

   where I is the intensity of the sound and I_0 is a constant. Find and interpret $f^{-1}(N)$.

Section 10.3 THE GRAPH, DOMAIN, AND RANGE OF AN INVERSE FUNCTION

1. Let $f(x) = 5^x$.
   (a) Find $f^{-1}(x)$
   (b) Graph $f(x)$ and $f^{-1}(x)$ on the same graph
   (c) Find the domain and range of $f(x)$ and $f^{-1}(x)$
2. Sketch a graph of the inverse function.



3. Check that the functions are inverses.
   (a) $f(x) = 1 + 7x^3$ and $f^{-1}(x) = \sqrt[3]{\frac{x-1}{7}}$
   (b) $g(x) = 1 - \frac{1}{x-1}$ and $g^{-1} = 1 + \frac{1}{x+1}$

Section 10.4 COMBINATIONS OF FUNCTIONS

1. For $f(x) = 4x^2 + 7$, $g(x) = \ln(x)$, $h(x) = \sin(x-3)$, and $m(x) = 3x+4$ find the following and simplify:

   (a) $\dfrac{f(x)}{g(x)}$

   (b) What is the domain of $\dfrac{f(x)}{g(x)}$?

   (c) $h(f(x))$

   (d) What is the domain of $h(f(x))$?

   (e) $m(f(x))$

   (f) $f(m(x))$

   (g) $f(x) \cdot m(x)$

   (h) $f(x) + m(x)$

   (i) $f(x) - m(x)$

2. Find simplified formulas using

$$u(x) = 2x - 1,\ v(x) = 1 - x,\ w(x) = \frac{1}{x}$$

   (a) $f(x) = u(x) + v(x)$

   (b) $g(x) = w(x) \cdot v(x)$

   (c) $h(x) = 2u(x) - 3v(x)$

   (d) $j(x) = \frac{u(x)}{v(x)}$

   (e) $k(x) = (v(x))^2$

   (f) $l(x) = u(x) - v(x) - w(x)$

3. Let $f(t)$ be the number of men and $g(t)$ be the number of women in Canada in year t. Let $h(t)$ be the average income, in Canadian dollars, of women in Canada in year t.

   (a) Find the function $p(t)$, which gives the number of people in Canada in year t.

   (b) Find the total amount of money $m(t)$ earned by Canadian women in year t.

4. An average of 50,000 people visit Riverside Park each day in the summer. The park charges \$15.00 for admission. Consultants predict that for each \$1.00 increase in the entrance price, the park would lose an average of 2500 customers per day. Express the daily revenue from ticket sales as a function of the number of \$1.00 price increases. What ticket price maximizes the revenue from ticket sales?

5. Use the table to make tables of values for $x = -1$, 0, 1, 2, 3, 4 for the following functions.

   (a) $h(x) = f(x) + g(x)$

   (b) $j(x) = 2f(x)$

   (c) $k(x) = (g(x))^2$

   (d) $m(x) = g(x)/f(x)$

x	-1	0	1	2	3	4
$f(x)$	-4	-1	2	5	8	11
$g(x)$	4	1	0	1	4	9